MATH3705 Tutorial 1

1. Find the Laplace transforms of the given functions.

(i)

$$f(t) = 6\mathbf{e}^{-5t} + \mathbf{e}^{3t} + 5t^3 - 9$$

Solution:

$$F(s) = 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\,\frac{3!}{s^{3+1}} - 9\,\frac{1}{s}$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s}$$

(ii)

$$g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$$

Sol:

$$G(s) = 4\frac{s}{s^2+(4)^2} - 9\,\frac{4}{s^2+(4)^2} + 2\frac{s}{s^2+(10)^2}$$

$$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}$$

(iii)

$$h(t) = 3\sinh(2t) + 3\sin(2t)$$

Sol:

$$H(s) = 3\frac{2}{s^2-(2)^2} + 3\,\frac{2}{s^2+(2)^2}$$

$$= \frac{6}{s^2-4} + \frac{6}{s^2+4}$$

(iv)

$$g(t) = t^{\frac{3}{2}}$$

Sol:

$$G(s) = \frac{3}{2}\left(\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}\right)\left(\frac{1}{s}\right) = \frac{3\sqrt{\pi}}{4s^{\frac{5}{2}}}$$

2. Find the inverse transform of each of the following.

(i)

$$F(s) = \frac{6}{s} - \frac{1}{s-8} + \frac{4}{s-3}$$

Sol:

$$F(s) = 6\,\frac{1}{s} - \frac{1}{s-8} + 4\,\frac{1}{s-3}$$

$$f(t) = 6(1) - e^{8t} + 4\left(e^{3t}\right)$$

$$= 6 - e^{8t} + 4e^{3t}$$

(ii)

$$H(s) = \frac{19}{s+2} - \frac{1}{3s-5} + \frac{7}{s^5}$$

Sol:

$$H(s) = \frac{19}{s-(-2)} - \frac{1}{3\left(s-\frac{5}{3}\right)} + \frac{7\,\frac{4!}{4!}}{s^{4+1}}$$

$$= 19\,\frac{1}{s-(-2)} - \frac{1}{3}\,\frac{1}{s-\frac{5}{3}} + \frac{7}{4!}\,\frac{4!}{s^{4+1}}$$

Thus

$$h(t) = 19e^{-2t} - \frac{1}{3}e^{\frac{5t}{3}} + \frac{7}{24}t^4$$

(iii)

$$F(s) = \frac{6s}{s^2+25} + \frac{3}{s^2+25}$$

Sol:

$$F(s) = 6\frac{s}{s^2 + (5)^2} + \frac{3\frac{5}{5}}{s^2 + (5)^2}$$

$$= 6\frac{s}{s^2 + (5)^2} + \frac{3}{5}\frac{5}{s^2 + (5)^2}$$

Thus

$$f(t) = 6\cos(5t) + \frac{3}{5}\sin(5t)$$